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Anna RUTKOWSKA-ZIARKO
University of Warmia and Mazury in Olsztyn
Przemysław GARSZTKA*
Poznań University of Economics

Diversification of risk of a fundamental portfolio based on semi-variance

Abstract: The following considerations are based on the concept of the fundamental portfolio as was proposed by [Tarczyński 1995]. In addition, in this article semi-variance, as an alternative to variance, was used as a measure of risk. The paper aims to propose and present empirical verification of the iterative algorithm for risk diversification in a fundamental portfolio with minimum semi-variance. The calculations were made assuming that we had a starting portfolio and that it could be modified to achieve the optimal solution under the established conditions The same calculations were performed for several starting portfolios.
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Introduction

The construction of a portfolio of securities in which the investment risk would be properly evaluated is a constant subject of discussion since the formulation of the problem of the Markowitz's portfolio [Markowitz 1952]. Currently most scientists agree that the downside risk measures are a good approach to assess the risk of the investment. One of the most popular methods of downside risk measurement in case of shares is still the use of semi-variance or semi-deviation [Wolski 2013; Pla-Santamaria & Bravo 2013]. This approach is well embedded in the theory of the construction of the portfolio, intuitive [Washer & Johnson 2013] and allows easy generalization in the context of the von Neumann-

* Corresponding author: przemyslaw.garsztka@ue.poznan.pl.

Morgenstern utility theory [Cumova & Nawrocki 2014]. In the literature can be also be found other methods of assessing the downside risk, such as the assessment of the Value at Risk (VaR) [Atligan & Demirtas 2013] or the use of VaR and models of conditional volatility GARCH type [Hammoundeh, Santos & Al-Hassan 2013] – to mention the latest works. However these methods are not recognised in relation to long-term investments in the stock market.

Another approach to controlling and reducing investment risk can be associated with the selection of shares for the portfolio related to their attractiveness for investors. It is an approach that allows for the addition of a third dimension analysis of the construction of the securities portfolio. Such an approach can be found in relation to individual indicators [Jacobs & Levy 2013] and synthetic measures of aggregate assembly of such indicators [Tarczyński 1995; Gazińska & Tarczyński 2011]. In the paper [Jacobs & Levy 2013] in the utility function the risks associated with leverage were taken into account. That function include the risks and costs of margin calls – which can force borrowers to liquidate securities at adverse prices due to illiquidity – losses exceeding the capital invested, and the possibility of bankruptcy.

The following considerations are based on the concept of the fundamental portfolio as was proposed by Tarczyński [1995]. Tarczyński applied a synthetically developed measurement to evaluate the economic and financial standing of a company as used for portfolio construction. He called this measure the *taxonomic measure of attractiveness of investment* – TMAI.

In recent years the Tarczyński model was modified by introducing, for example, a measure of risk into the objective function and by taking into account connections between the profitability of shares [Rutkowska-Ziarko 2011]. Due to a possible correlation between diagnostic financial variables the Mahalanobis distance was used to determine the TMAI [Rutkowska-Ziarko 2013].

In addition, in this semi-variance, as an alternative to variance, was used as a measure of risk. Risk in existing models of the fundamental portfolio was measured by variance [Tarczyński 1995; Tarczyński & Łuniewska 2005; Rutkowska-Ziarko 2011, 2013].

One of the drawbacks of variance as a measure of risk is that negative and positive deviations from the expected rate of return are treated in the same manner. In fact negative deviations are undesirable, whilst positive ones create an opportunity for a higher profit. Determining effective fundamental portfolios for semi-variance is more complicated than for variance because the parameters in the objective function for semi-variance depend on the portfolio composition.

The paper aims to propose and present empirical verification of the iterative algorithm for risk diversification in the fundamental portfolio with minimum semi-variance.

The main aim of the article is to propose a method which will allow the composition of a fundamental portfolio with minimum semi-variance. An additional objective is to analyze the impact, by taking into account the financial and economic standing of the company in the portfolio selection model, on the efficient frontier. In terms of application the aim of the article is to verify the convergence of the proposed method.

1. Taxonomic measure of attractiveness of investments

Four financial ratios were taken as diagnostic variables in this paper. Three of them described the financial situation of the companies under study: quick ratio (*CR*), return on assets ratio (*ROA*) and debt ratio (*DR*). The study also took the market price-earnings ratio (*P/E*) into account. Studies of capital markets have revealed a negative correlation between the value and future share price increases [Basu 1977]. Therefore, *P/E* was regarded as a destimulant and replaced with E/P:

$$E/P = \frac{1}{P/E}. \tag{1}$$

The ROA ratio was regarded as a stimulant, whereas DR as a destimulant. DR was replaced with a corresponding stimulant (*DR'*).

$$DR' = \frac{1}{DR}. \tag{2}$$

In the case of liquidity ratios the high values are considered to be a symptom of maintaining too high a capital [Tarczyński 2002]. On the other hand loss of liquidity is often more serious for the company than the lack of profitability [Szczepaniak 1996; Sierpińska & Wędzki 1997]. Therefore the reference value of the liquidity ratio can be regarded as a certain level which is neither too small nor too large. The quick ratio as the stimulant to the level of 1 was used. QR was replaced with QR', according to the formula:

$$QR' = \begin{cases} 1 & for \quad QR \geq 1 \\ QR & for \quad QR < 1 \end{cases}. \tag{3}$$

The author assumed that if the company had the quick ratio at level 1 then it had good liquidity and that higher liquidity was neither superior nor inferior.

The quick ratio (CR) indicates the degree of coverage of short-term foreign capital by current assets with a high degree of liquidity. In the literature an indicator value of 1.0 was considered to be satisfactory [Gąsiorkiewicz 2011, p. 58], although some authors have suggested keeping it at a higher level [Zelek 2003, p. 94]. Studies on the Warsaw Stock Exchange showed that investors preferred companies with high rates of profitability and high levels of cash [Bolek & Wolski 2012].

The Mahalanobis distance was used to determine the taxonomic measure of attractiveness of investments for each company.

Let w_{il} denote values of diagnostic variables after transformation of the variables into stimulants, where $l = 1, \ldots, m$ is the number of diagnostic variables considered. The highest observed value of w_{0l} is sought for each diagnostic variable [Hellwig 1968, pp. 323–326]:

$$w_{0l} = \max_i \{w_{il}\}. \tag{4}$$

An abstract point $P_0(w_{0l})$ was taken as the reference standard; its coordinates assume the highest values of the diagnostic variables after transformation of the variables into stimulants.

The Mahalanobis distance could be calculated as follows [Mahalanobis 1936, p. 50]:

$$MQ_i = \sqrt{(W_i - W_l) \cdot C^{-1} (W_i - W_l)^T}, \tag{5}$$

where W_i is a row vector, $W_i = [w_{11}, \ldots, w_{i4}]$, W_l is a row vector representing "the ideal quoted company" $W_l = [w_{01}, \ldots, w_{04}]$, and C is the covariance matrix for diagnostic variables.

The Mahalanobis distance was used to determine the taxonomic measure of attractiveness of investments for each company [Tarczyński 2002, p. 98]:

$$TMAI_i = 1 - \frac{Q_i}{\max_i \{Q_i\}}. \tag{6}$$

2. The fundamental portfolio and semi-variance

The concept of the fundamental portfolio combines two important approaches to financial investment, namely the portfolio theory and fundamental analysis. The model of the construction of the fundamental portfolio was proposed by Tarczyński [1995].

In the Tarczyński model the weighted sum of TMAI was maximized in the objective function. The risk was counted as a limitation and was the sum of the risks of the portfolio components not the risk of the whole portfolio [Rutkowska-Ziarko 2011].

The model of constructing a fundamental portfolio which was used in the study is a modification of the classic Markowitz model.

The classic Markowitz model [Markowitz 1952] is:

$$\min S_p^2 = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j \operatorname{cov}_{ij} \tag{7}$$

with the following limitations:

$$\sum_{i=1}^{k} x_i = 1, \tag{8}$$

$$\sum_{i=1}^{k} x_i \overline{z}_i \geq \gamma, \tag{9}$$

$$x_i \geq 0 \; i = 1, \ldots, k. \tag{10}$$

It was modified by introducing the additional condition by taking into account only firms that possess a good financial and economic standing [Rutkowska-Ziarko 2011]:

$$TMAI_p = \sum_{i=1}^{k} TMAI_i x_i \geq TMAI_\gamma, \tag{11}$$

where: S_p^2 is the variance of rate of return; cov_{ij} is covariance between security i and security j, γ is the target rate of return assuming that $\gamma \leq \max \overline{z}_i$; $\overline{z}_i$ is the mean rate of return on security i; x_i is the contribution by the value of the i-th share in the portfolio; and $TMAI_\gamma$ is the sum of TMAI, as required by the investor which is weighted by the contribution of the shares in the portfolio.

A limiting condition was introduced to the portfolio construction model according to which the TMAI total weighted by the contribution of shares of a specific company in the portfolio must achieve at least the level set by the investor.

Considering the drawbacks of variance as a measure of risk, a monograph on the choice of portfolio by Markowitz [Markowitz 1959] suggested semi-variance of the assumed rate of return $dS^2(\gamma)$ as a measure of risk which is an alternative to variance:

$$dS^2(\gamma) = \frac{\sum_{t=1}^{m} d_t^{\,2}(\gamma)}{m-1}, \; t = (1, 2, \ldots, m), \tag{12}$$

where:

$$d_t(\gamma) = \begin{cases} 0 & for \quad z_t \geq \gamma \\ z_t - \gamma & for \quad z_t < \gamma \end{cases}. \tag{13}$$

When the semi-variance of an investment portfolio is determined, semi-covariances of the rates of return shares of which it comprises are used

$$dS_p^2(\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j d_{ij}(\gamma), \tag{14}$$

where: $dS_p^2(\gamma)$ is the semi-variance of the portfolio rates of return; and $d_{ij}(\gamma)$ is semi-covariance of the rate of return for the i-th and the j-th share.

When semi-covariance is determined it is noted in which periods the rate of return is higher and in which periods it is lower than the level assumed by the investor:

$$d_{ij}(\gamma) = \frac{1}{m-1}\sum_{t=1}^{m} d_{ijt}(\gamma), \tag{15}$$

where:

$$d_{ijt}(\gamma) = \begin{cases} 0 & dla \;\; z_{pt} \geq \gamma \\ (z_{it} - \gamma)(z_{jt} - \gamma) & dla \;\; z_{pt} < \gamma \end{cases}, \tag{16}$$

where:

$$z_{pt} = \sum_{i=1}^{k} x_i z_{it}, \; t = (1, 2, \ldots, m). \tag{17}$$

Determining effective portfolios for the risk understood as denotied by the possibility of achieving a lower rate of return than the assumed value is reduced to minimizing the semi-variance of the assumed rate of return at the predetermined value of g, therefore, to solving the following optimizing problem:
minimize the semi-variance of the portfolio rate of return:

$$dS_p^2(\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j d_{ij}(\gamma),$$

with the limitations (8–11).

Furthermore in the article the fundamental portfolio with minimum variance will be referred to as VFP whilst that with minimum semi-variance as SFP.

Using semi-variance to determine effective portfolios creates considerable problems. When the semi-covariances of rates of return are determined $d_{ij}(\gamma)$ one has to know in which periods the rate of return of the entire portfolio was lower than the assumed value and this depends both on the assumed rate of return and on the portfolio composition. This makes determining effective portfolios for semi-variance of the assumed rate of return more complicated than for variance. When a portfolio with minimum semi-variance is determined each time the composition of the portfolio or the assumed rate of return g changes the semi-covariances of the rates of return $d_{ij}(\gamma)$ should be re-estimated.

The iterative algorithm was used in order to determine an effective fundamental portfolio which would minimize semi-variance of the target rate of return. A modification of the iterative algorithm is used to build a portfolio with the minimum semi-variance that was proposed by Rutkowska-Ziarko [2005].

Starting with the VFP portfolio the following procedure is reiterated until self-stabilization[1] of the semi-variance of the portfolio has been achieved:

1. **Determining the rates of return of portfolio z_{pt} within time units according to (16).**
2. **Determining the semi-covariances of rates of return $d_{ij}(\gamma)$ (15–16).**
3. **For the semi-covariance of rates of return $d_{ij}(\gamma)$ as determined in point 2 – minimize the semi-variance of portfolio rate of return:**

[1] Self-stabilization is understood as the stabilization of the portfolio composition at a set level of precision.

$$ds_p^2(\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j d_{ij}(\gamma),$$

with the limitations (8–11).

In this study we used **R** – an environment for statistical computing to solve the problem.

3. Empirical results

The study covered 20 companies listed on The Warsaw Stock Exchange (included in the WIG20 index and 10 WIG80), and excluding financial institutions.[2] The study was based on the quarterly rates of return calculated based on daily closing prices during the period of from January 1st, 2011 to March 22nd, 2012. The rates of return were computed as relative increases in the prices of stocks according to the formula:

$$R_{it} = \frac{N_{i,t+s} - N_{it}}{N_{it}} \cdot 100\%, \tag{18}$$

where R_{it} is the rate of return on security i at time t, s is the length of the investment process expressed in days, N_{it} is the listed value of the security i at time t, and $N_{i,t+s}$ is the listed value of the security i after s days of investing started at time t.

The share closing price on March 2nd2, 2012 was taken as a company's market share price. Financial ratios were calculated for each company based on their annual financial reports for 2011.

Efficient fundamental portfolios with minimum variance values (VFP) as well as fundamental portfolios with minimum semi-variance values (SFP) were built for selected levels of the target rate of return (γ) and $TMAI_\gamma = 0.3; 0.4; 0.5$. Accuracy of the calculations was estimated up to 8 digits after the decimal point. The analysis started by finding the SFP from a different starting point: VFP, an equally weighted portfolio and the Markowitz portfolio with the condition that the average rate of return should be positive.

To present the proposed algorithm, $\gamma = 2\%$ and $TMAI_\gamma = 0.4$ were used, then iterations 1 to 10 were analyzed. The composition of the determined portfolios (by value) and their selected characteristics are both presented in Tables 1–3.

[2] The three-letter abbreviations used at the Warsaw Stock Exchange are used in the paper instead of the full names of the stock issuers.

Table 1. Finding the SFP portfolio starting from VFP for γ = 2% and $TMAI_\gamma$ = 0.4

Issuer	Starting portfolio: VFP	Portfolio composition in the i-th iteration			
		1	**2**	**3**	**… 10**
KGH	0.000000029	0.000000028	0.000000028	0.000000028	0.000000028
PKN	0.000000020	0.000000020	0.000000020	0.000000020	0.000000020
PGE	0.000000001	0.000000001	0.000000001	0.000000001	0.000000001
PGN	0.000000050	0.000000050	0.000000050	0.000000050	0.000000050
TPE	0.000000099	0.000000099	0.000000099	0.000000099	0.000000099
LWB	0.000000007	0.000000007	0.000000007	0.000000007	0.000000007
TPS	0.000000008	0.000000008	0.000000008	0.000000008	0.000000008
KER	0.000000010	0.000000010	0.000000010	0.000000010	0.000000010
ACP	0.000000052	0.000000052	0.000000052	0.000000052	0.000000052
SNS	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
KGN	0.000000027	0.000000027	0.000000027	0.000000027	0.000000027
AMC	0.000000011	0.000000011	0.000000011	0.000000011	0.000000011
DUD	0.000000052	0.000000052	0.000000052	0.000000052	0.000000052
ASE	0.301083505	0.415288985	0.401747825	0.401663436	**0.4012**
ACT	0.000000035	0.000000035	0.000000035	0.000000035	0.000000035
GNT	0.166481387	0.068154854	0.079813273	0.079885929	**0.0803**
ACE	0.369701678	0.361052164	0.362077724	0.362084115	**0.3621**
MCR	0.000000066	0.000000066	0.000000066	0.000000066	0.000000066
PUE	0.000000154	0.000000154	0.000000154	0.000000154	0.000000154
LTX	0.162732808	0.155503376	0.156360558	0.156365900	**0.1564**
	risk, profitability and TMAI in the i-th iteration				
	starting portfolio	1	2	3	… 10
Average rate of return	2.0	2.0	2.0	2.0	2.0
Variance	43.2465	45.625	45.0944	45.0913	45.0754
Semi-variance of 2%	26.0625	25.5572	25.5437	25.5437	25.5437
$TMAI_\gamma$	0.40	0.40	0.40	0.40	0.40

Source: Authors' own calculations.

Table 2. Finding the SFP portfolio starting from an equally weighted portfolio, for γ = 2% and $TMAI_\gamma$ = 0.4

Issuer	**Starting portfolio: equally weighted**	**Portfolio composition in the i-th iteration**			
		1	**2**	**3**	**… 10**
KGH	0.05	0.000000027	0.000000027	0.000000026	0.000000026
PKN	0.05	0.000000014	0.000000014	0.000000014	0.000000013
PGE	0.05	0.000000000	0.000000000	0.000000000	0.000000000
PGN	0.05	0.000000044	0.000000044	0.000000043	0.000000042
TPE	0.05	0.000000058	0.000000058	0.000000056	0.000000056
LWB	0.05	0.000000006	0.000000006	0.000000006	0.000000005
TPS	0.05	0.000000007	0.000000007	0.000000007	0.000000007
KER	0.05	0.000000006	0.000000006	0.000000005	0.000000005
ACP	0.05	0.000000034	0.000000034	0.000000034	0.000000033
SNS	0.05	0.000000003	0.000000003	0.000000003	0.000000003
KGN	0.05	0.000000033	0.000000033	0.000000032	0.000000032
AMC	0.05	0.000000009	0.000000009	0.000000008	0.000000008
DUD	0.05	0.000000044	0.000000044	0.000000043	0.000000043
ASE	0.05	0.236859978	0.451435989	0.405785177	**0.4007**
ACT	0.05	0.000000030	0.000000030	0.000000029	0.000000029
GNT	0.05	0.221775378	0.037033671	0.076337279	**0.0807**
ACE	0.05	0.374565893	0.358314674	0.361772118	**0.3621**
MCR	0.05	0.000000056	0.000000056	0.000000054	0.000000053
PUE	0.05	0.000000082	0.000000082	0.000000080	0.000000079
LTX	0.05	0.166798297	0.153215212	0.156104987	**0.1564**
	risk. profitability and TMAI in the i-th iteration				
	starting portfolio	1	2	3	… 10
Average rate of return	−4.4147	2.0	2.0	2.0	2.0
Variance	146.8334	43.9987	47.3688	45.2456	45.0576
Semi-variance of 2%	161.9681	26.8663	25.7146	25.5454	25.5436
$TMAI_\gamma$	0.3637	0.40	0.40	0.40	0.40

Source: Authors' own calculations.

Table 3. Finding the SFP portfolio starting from an equally weighted portfolio, for γ = 2% and $TMAI_\gamma$ = 0.4

Issuer	**Starting portfolio: Markovitz**	**Portfolio composition in the i-th iteration**			
		1	**2**	**3**	**4**
KGH	0.00000009	0.00000009	0.00000009	0.00000009	0.00000009
PKN	0.0	0.0	0.0	0.0	0.0
PGE	0.0	0.0	0.0	0.0	0.0
PGN	0.00000006	0.00000006	0.00000006	0.00000006	0.00000006
TPE	0.00000003	0.00000003	0.00000003	0.00000003	0.00000003
LWB	0.02631154	0.00000096	0.00000096	0.00000096	0.00000096
TPS	0.00000006	0.00000006	0.00000006	0.00000006	0.00000006
KER	0.00000001	0.00000001	0.00000001	0.00000001	0.00000001
ACP	0.0	0.0	0.0	0.0	0.0
SNS	0.00000005	0.00000004	0.00000004	0.00000004	0.00000004
KGN	0.00000006	0.00000006	0.00000006	0.00000006	0.00000006
AMC	0.0	0.0	0.0	0.0	–
DUD	0.00000003	0.00000003	0.00000003	0.00000003	0.00000003
ASE	0.33426553	0.35135890	0.36685638	0.36685672	**0.3669**
ACT	0.00000004	0.00000004	0.00000004	0.00000004	0.00000004
GNT	0.07358228	0.12320250	0.10985977	0.10985948	**0.1099**
ACE	0.47731153	0.36587552	0.36470180	0.36470177	**0.3647**
MCR	0.08314376	0.00000325	0.00000325	0.00000325	0.00000325
PUE	0.00000002	0.00000002	0.00000002	0.00000002	0.00000002
LTX	0.00538490	0.15955841	0.15857739	0.15857737	**0.1586**
	risk. profitability and TMAI in the i-th iteration				
	starting port-folio	1	2	3	… 10
Average rate of return	0.2720	2.0	2.0	2.0	2.0
Variance	31.7976	43.7087	44.0366	44.0366	44.0366
Semi-variance of 2%	27.1179	25.6774	25.6076	25.6076	25.6076
$TMAI_\gamma$	0.3478	0.40	0.40	0.40	0.40

Source: Authors' own calculations.

It is apparent that the proposed procedure allowed us to find a portfolio with lower semi-variance as compared to the starting portfolio. In subsequent iterations semi-variance was decreased sometimes at the expense of increasing variations. Other characteristics of the portfolio, e.g. the average rate of return and TMAI were at the same level. The largest decrease in semi-variance took place in the first iteration. The limitation concerning the required level of TMAI was an active limitation.

Other portfolios were used as a starting point to check the convergence of the algorithm in this case. Starting from the equally weighted portfolio almost the same solution was obtained.

Starting from the equally weighted portfolio almost the same solution was obtained. In two of the discussed cases there were no significant differences in the structure and characteristics of the portfolios between iteration 3 and 10.

For the Markowitz portfolio we found the problem with the convergence of the algorithm as a starting point. The procedure stopped after the fourth iteration – there was a technical problem with the solver. The given solution was not as good as it had been earlier. We could see no significant changes in the portfolio structure between iterations 2 and 3 and there was no decrease of semi-variance.

We constructed efficient frontiers for various TMAI values in order to analyze the impact that the company's financial and economic standing had

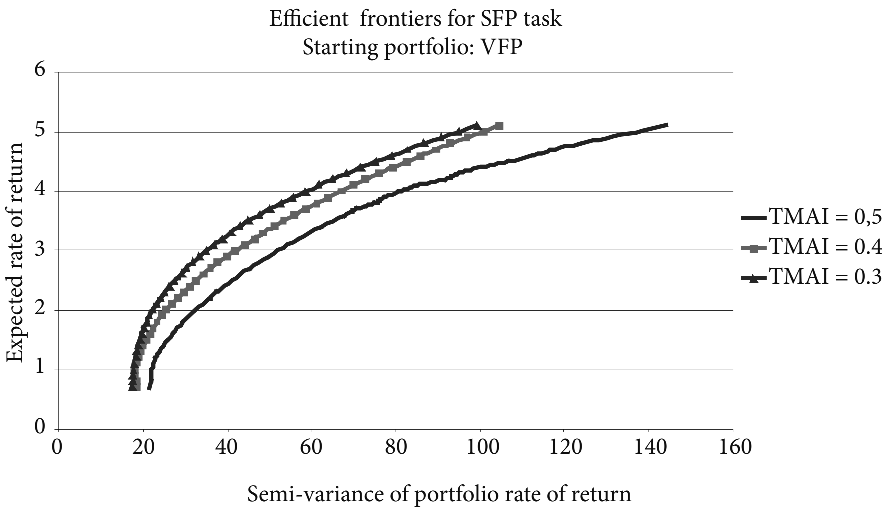

Figure 1. Efficient frontiers starting from VFP for cases of $TMAI_\gamma$ = 0.3; 0.4; 0.5
Source: Authors' own calculations

on the shape of the efficient frontier during portfolio optimization. We used the proposed procedure in order to find efficient fundamental portfolios for semi-variance. The VFP was used as the starting point for 50 different target rates of return. Figure 1 shows the efficient frontiers for the SFV task for cases of $TMAI_{\gamma} = 0.3; 0.4; 0.5$.

One can see that the higher level of the TMAI moves down the efficient frontier. In all cases the limitation concerning the required level of TMAI was an active limitation.

Other portfolios were used as the starting points to check the convergence of the algorithm. Very similar efficient frontiers were obtained for 50 different rates of return as in Figures 2–4.

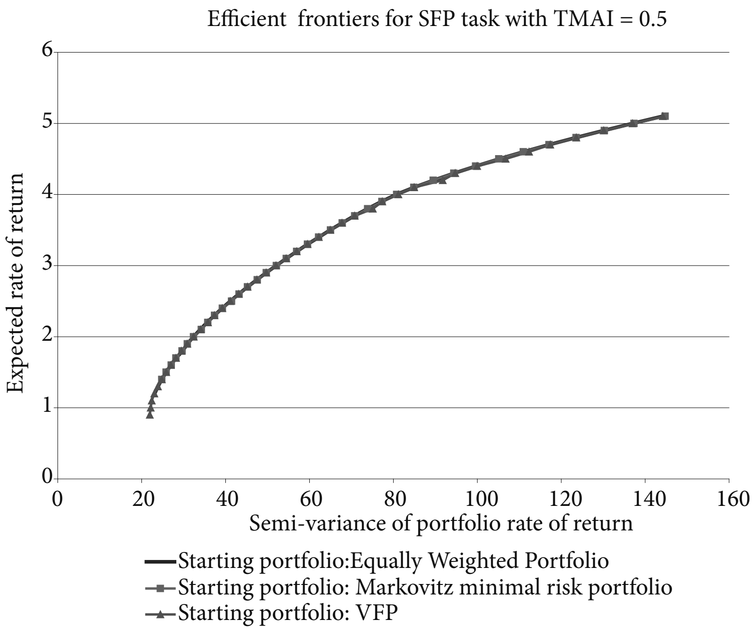

Figure 2. Efficient frontiers for $TMAI_{\gamma}$ = 0.5
Source: Authors' own calculations

Efficient frontiers of the portfolios that were calculated based on different starting points were very similar but not identical. Problems may occur in calculations for low expected rates of return due to the very complicated procedure of calculating SFP portfolios.

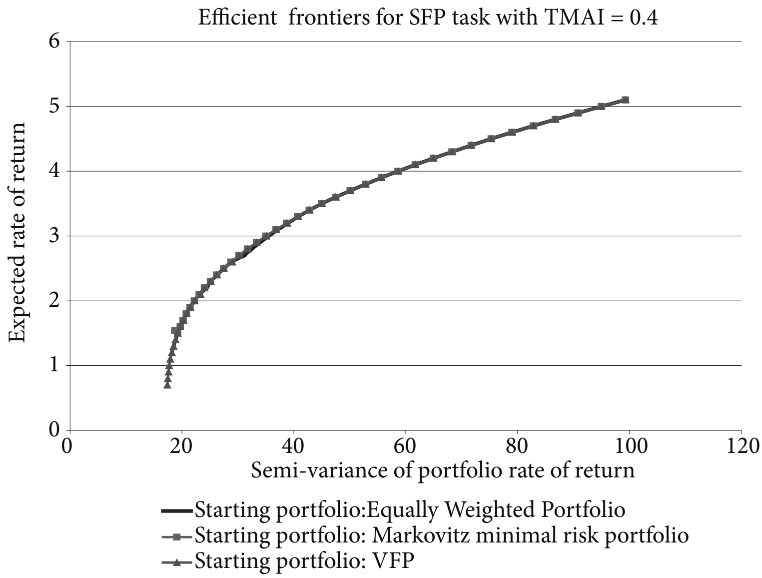

Figure 3. Efficient frontiers for $TMAI_{\gamma}$ = 0.4
Source: Authors' own calculations

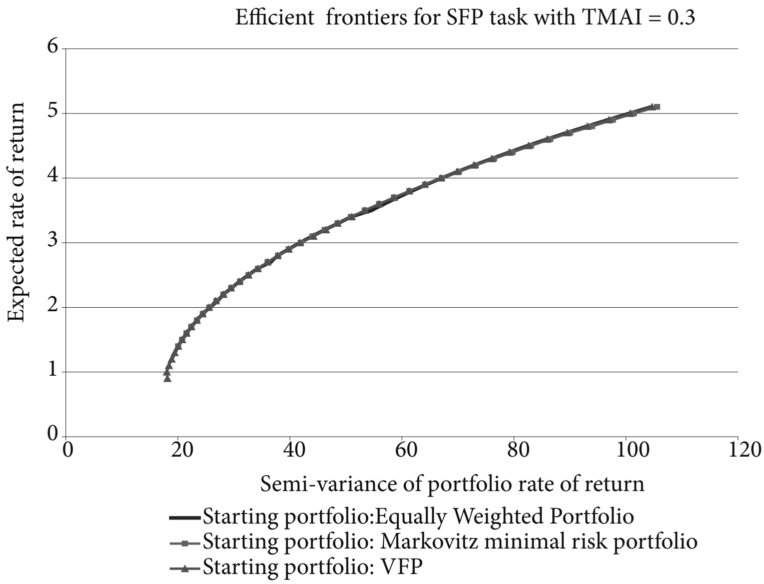

Figure 4. Efficient frontiers for $TMAI_{\gamma}$ = 0.3
Source: Authors' own calculations

Conclusions

In this paper the authors proposed an algorithm to construct a fundamental portfolio of securities by assuming that semi-variance is an appropriate measure of investment risk. Moreover, it was empirically verified that in the case of optimal portfolios due to the variance of the rate of return it is possible to further reduce the investment risk when taking into account semi-variance. It was observed that, in comparison to the approaches that have been used so far in the literature, the proposed algorithm leads to safer portfolios in the context of downside risk.

The calculations were made assuming that we had a starting portfolio and that it could be modified to achieve the optimal solution under the established conditions The same calculations were performed for several starting portfolios. The most stable calculation procedure was observed for VFP as a starting point. It was also observed that the first iteration was the most important.

During the calculations the authors had some trouble with portfolios that were close to minimal variation. The portfolios were unstable in cases where the expected rate of return was below a minimal positive rate of return taken from the shares. This could be a technical problem.

Introducing another condition of the TMAI into the portfolio selection model has an influence on the efficient frontier as the higher level of TMAI moves down the efficient frontier. In all cases the limitation concerning the required level of TMAI was an active limitation.

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