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The quantity and quality of human capital in Poland and USA analyzed in the framework of the Manuelli-Seshadri model

Abstract: The article is aimed at analyzing the quantity and quality of human capital in Poland as compared with the United States and other countries in the framework of the recent model by Manuelli and Seshadri. A description of the model is given. The analyzed quantities are: the stock of human capital, expenditures and time allocated to human capital formation, the wage rate per unit of human capital (a measure of the quality of human capital which is a central concept of the model) and GDP *p.c.* Possible modifications of the model have been suggested.

Keywords: human capital, quality and quantity of human capital, Manuelli-Seshadri model.

JEL codes: 041, 047.

1. Introduction

The article is aimed at analyzing the quantity and quality of human capital in Poland as compared with the United States and other countries in the framework of the recent model by Manuelli and Seshadri (2005).

The model is somewhat different from other human capital models[1]. First and foremost, the notion of the quality[2] of human capital has been introduced. The need for analyzing the quality of human capital, in addition to its quantity, comes from

[1] A review of human capital models is given e.g. in: Cichy (2005) (in Polish).

The most classic human capital models can be found particularly in: Ben-Porath (1967), Lucas (1988), Rebelo (1991), Mankiw, Romer, Weil (1992), Jones (1996).

[2] Before Manuelli and Seshadri, many researchers have postulated that the quality of schooling might influence human capital as a production factor, e.g. Barro (2001).

the fact that e.g. 10 years of schooling in highly developed countries is in no way equivalent to 10 years of schooling in African or Latin American countries. The persons educated in the two types of countries have the same quantity of human capital, but the quality differs substantially, which results in very disparate efficiencies of these persons as factors of production. The models of human capital as an economic growth factor should take this distinction into consideration.

In the model under examination, the acquisition of human capital is modeled as part of the standard income maximization problem[3], following the classic work of Ben-Porath (1967). The most important alteration with regard to the earlier models is the way human capital is measured. In the majority of models, discrepancies in the quality of human capital are not taken into account. Thus, human capital as an economic growth factor cannot account for cross-country differences in output *per capita*.

Manuelli and Seshadri have calibrated the model to match some empirical facts about the US economy *circa* 2000, which enables us to draw comparisons with this economy. Some parameters, e.g. physical capital depreciation rate or interest rate have been chosen arbitrarily, following standard values used in the literature.

In this paper we will apply the Manuelli-Seshadri model to estimate the human capital stock in Poland and compare it with the Authors' results for the United States and other countries. We will also attempt to indicate the possible modifications of the model so as to improve its value and quality.

2. Setup of the Manuelli-Seshadri model

2.1. Parameters

B – age when children of the representative individual are born,
R – retirement age of the representative individual,
T – length of life of the representative individual,
r – interest rate,
δ_k – physical capital depreciation rate,
δ_h – human capital depreciation rate,
z, θ – production function parameters[4],
z_h, γ_1, γ_2 – human capital production function parameters,
h_B, ν – "early human capital" production function parameters.

[3] In an earlier version of the model, human capital acquisition was modeled as part of the utility of consumption maximization problem for a representative household. The Authors abandoned this approach in favor of the income maximization formulation, efficiently equivalent to the former but much simpler.

[4] The parameter ***z*** is called Total Factor Productivity (TFP).

2.2. Variables

a – age of the representative individual under consideration,
$h(a)$ – human capital stock of an individual at age a,
w – wage rate per unit of human capital,
$x(a)$ – expenditures on market goods allocated to the production of human capital of an individual at age a,
x_E – parents' investments in early childhood of their children (determining the stock of human capital at age 6 h_E),
$n(a)$ – time allocated to human capital formation at age a,
$1\text{-}n(a)$ – time allocated to work at age a,
f – natural logarithm of the number of children that are born in the representative individual's household,
η – growth rate of population (= f/B),
s – years of schooling,
$\bar{h}$ – level of human capital *per capita* in equilibrium,
k – physical capital *p.c.*,
p_k – relative price of physical capital (p_k = 1 in USA),
κ – physical capital – human capital ratio,
y – output (GDP) *p.c.*

2.3. Income maximization problem

$$\max \int_6^R e^{-r(a-6)} \left(wh(a)(1-n(a)) - x(a) \right) da - x_E \qquad (1)$$

subject to the human capital dynamics equation:

$$\dot{h}(a) = z_h \left(n(a)h(a) \right)^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a),\ a \in [6,\ R), \qquad (2)$$

and the technology of early childhood human capital production:

$$h(6) \equiv h_E = h_B x_E^{\nu}. \qquad (3)$$

The representative individual chooses the time and expenditures on market goods allocated to the formation of human capital, thus determining its optimal (discounted net income maximizing) level. According to Eq. (2), the change in the stock of human capital is positive whenever its production (the first expression on the right-hand side of (2)) exceeds its depreciation (the second expression). The initial value of the stock of human capital (when the child begins its formal education at age 6) is determined by the amount of the parent's expenditures on medical care, nutrition, development of learning skills etc. between age 0 and 6.

2.4. The optimal path of human capital growth and conclusions from the model for the US economy

It can be shown that the optimal, income-maximizing path of human capital growth satisfies the following equations[5]:
– the initial level of human capital (age 6):

$$h_E = \left(\nu^\nu h_B \left(\frac{\gamma_1^{\gamma_1(1-\gamma_2)} \gamma_2^{\gamma_1\gamma_2} z_h^{\gamma_1} w^{(1-\gamma_1)(1-\gamma_2)}}{(r+\delta_h)^{1-\gamma_2}} \right)^{\frac{\nu}{1-\gamma}} e^{-\nu(r+\delta_h(1-\gamma_1))s} m(6+s)^{\frac{\nu(1-\gamma_2)}{1-\gamma}} \right)^{\frac{1}{1-\nu(1-\gamma_1)}} ; \quad (4)$$

– the stock of human capital during the schooling period – age $a \in [6, 6+s)$:

$$h(a) = h_E e^{-\delta_h(a-6)} \left\{ 1 + \left(h_E^{-(1-\gamma)} \left(\frac{w}{r+\delta_h} m(6) \right)^{\gamma_2} \gamma_2^{\gamma_2} z_h \right)^{\frac{1}{1-\gamma_2}} \right.$$

$$\left. \times \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + \delta_h(1-\gamma_1)} \left(e^{\frac{\gamma_2 r + \delta_h(1-\gamma_1)}{1-\gamma_2}(a-6)} - 1 \right) \right\}^{\frac{1}{1-\gamma_1}} ; \quad (5)$$

– the stock of human capital during the post-schooling period – age $a \in [6+s, R]$:

$$h(a) = \left(\frac{\gamma_2^{\gamma_2} \gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta_h)^{\gamma}} \right)^{\frac{1}{1-\gamma}} \{ e^{-\delta_h(a-s-6)} \frac{\gamma_1}{r+\delta_h} m(6+s)^{\frac{1}{1-\gamma}} +$$

$$+ \frac{e^{-\delta_h(a-R)}}{\delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} dx \left(1 - x^{\frac{r+\delta_h}{\delta_h}} \right)^{\frac{\gamma}{1-\gamma}} ; \quad (6)$$

where:

$$m(a) = 1 - e^{-(r+\delta_h)(R-a)}. \quad (7)$$

The evolution of the stock of human capital for a hypothetical representative individual in the USA is shown in the Figure 1:

During the schooling period (between the age of 6 and 6+*s*; the average number of years of schooling *s* = 12,08) the stock of human capital increases rapidly. After

[5] The proof can be found in the original paper by Manuelli and Seshadri.

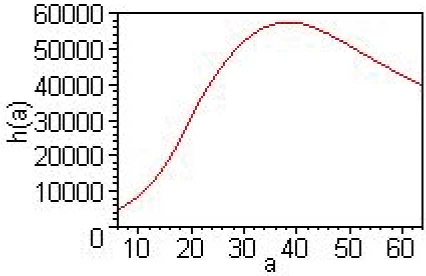

Figure 1. The optimal path of human capital growth for a representative individual in USA, aged 6 to 64
Source: Own calculations

this period the growth rate of human capital stock is gradually declining to zero around the middle of the period between age 6+*s* and *R*, when the depreciation of human capital starts to overcome its production. From this moment on, the level of human capital decreases until retirement (age *R*). The path of human capital stock in other countries (i.e. when the values of the model parameters and variables is different) is qualitatively similar.

The time allocated to human capital accumulation satisfies the following equations:

– during the schooling period – $a \in [6, 6+s)$:

$$n\,(a) = 1, \tag{8}$$

– during the post-schooling period – $a \in [6+s, R]$:

$$n(a) = \frac{m(a)^{\frac{1}{1-\gamma}}}{e^{-\delta_h(a-s-6)} m(6+s)^{\frac{1}{1-\gamma}} + \frac{(r+\delta_h)e^{-\delta_h(a-R)}}{\gamma_1 \delta_h} \int_{e^{\delta_h(6+s-R)}}^{e^{\delta_h(a-R)}} dx \left(1 - x^{\frac{r+\delta_h}{\delta_h}}\right)^{\frac{\gamma}{1-\gamma}}}. \tag{9}$$

The evolution of the fraction of time allocated to human capital formation ($n(a)$) for a hypothetical representative individual in USA is shown in the Figure 2:

During the formal education period the representative individual allocates all of their time to human capital accumulation. After the completion of this period, the individual devotes successively less time to human capital formation, the time going down to zero at the moment when the period of time left to retirement is so short that the production of human capital is no longer worthwhile (the foregone

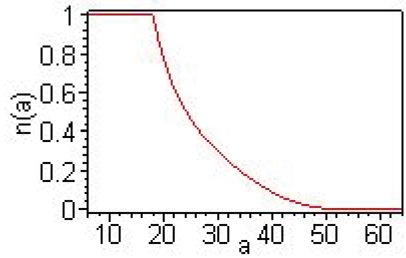

Figure 2. The optimal path of the fraction of time allocated to human capital formation for a representative individual in USA, aged 6 to 64
Source: Own calculations

earnings are higher than the potential earnings increase in the future due to bigger stock of human capital).

The last set of equations gives the amount of expenditures on market goods allocated to the production of human capital):
– during the schooling period – $a \in [6, 6+s)$:

$$x(a) = \frac{\gamma_2 w}{r+\delta_h}\left(\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta_h)^{\gamma}}\right)^{\frac{1}{1-\gamma}} e^{\frac{r+\delta_h(1-\gamma_1)}{1-\gamma_2}(a-s-6)} m(6+s)^{\frac{1}{1-\gamma}}; \tag{10}$$

– after the schooling period – $a \in [6+s, R]$:

$$x(a) = \frac{\gamma_2 w}{r+\delta_h}\left(\frac{\gamma_2^{\gamma_2}\gamma_1^{\gamma_1} z_h w^{\gamma_2}}{(r+\delta_h)^{\gamma}}\right)^{\frac{1}{1-\gamma}} m(a)^{\frac{1}{1-\gamma}}. \tag{11}$$

The following chart shows the expenditures of a representative individual in the USA:

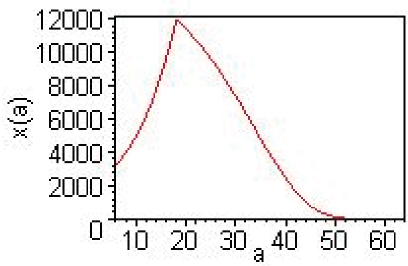

Figure 3. The amount of expenditures on market goods allocated to the production of human capital for a representative individual in USA, aged 6 to 64
Source: Own calculations

During the schooling period the amount of expenditures rises, reaching the maximum at age 6+*s*. After this period the human capital formation expenses diminish and reach zero when the production of human capital is no longer taken up.

2.5. Demographics

It is assumed in the model that each individual has e^f children[6], born at age *B*. The age distribution of the economy under consideration is given by the following equation:

$$N(a,t) = \phi(a)e^{\eta t}, \tag{12}$$

where:

$$\phi(a) = \eta \frac{e^{-\eta a}}{1-e^{-\eta T}}. \tag{13}$$

The number of persons between age *a* and $a+\delta a$ at time *t* equals $\int N(a',t)da'$ from *a* to $a+\delta a$.

Having determined the age structure of the population, we can find the average stock of human capital per person and thus calculate the *per capita* output of the economy.

2.6. Average stock of human capital per capita

The average stock of human capital per person is given by:

$$\bar{h} = \frac{\int_{6+s}^{R} h(a)(1-n(a))\phi(a)da}{\int_{6+s}^{R} \phi(a)da}. \tag{14}$$

It is determined by the path of human capital growth, the fraction of time allocated by the representative individual to work and their expenditures on market goods allocated to human capital formation.

2.7. Equilibrium

We consider the steady state given by the following set of equations (15)-(17).

[6] If every person has e^f children, then the growth rate of population equals *f/B*.

$$p_k(r+\delta_k)=F_k(\kappa,1), \tag{15}$$

where: $F(\kappa,1)$ – the production function (κ is the physical capital – human capital ratio), F_k – the derivative of the production function with respect to physical capital *p.c.*, and:

$$w=F_h(\kappa,1), \tag{16}$$

where: F_h – the derivative of the production function with respect to human capital *p.c.*

Equations (15), (16) lead to the following expression for output per worker:

$$y=F(\kappa,1)\bar{h}. \tag{17}$$

We assume equilibria on the physical capital market (marginal cost of physical capital equals its marginal productivity – Eq. (15)) and the labor market (marginal cost of human capital (the wage rate per unit of human capital) equals its marginal productivity – Eq. (16)). Such an approach seems reasonable for established market economies but there are a few doubts whether it is acceptable for countries like Poland, which are shortly after or during the transition period, or for poor countries whose economies differ substantially from the economies of highly developed countries. The results obtained from the model should be interpreted with these restrictions kept in mind.

3. Calibration of the parameters of the model

The model has been calibrated so that its steady state implications are consistent with some empirical observations for the US economy around 2000.

The production function is assumed to be of Cobb-Douglas form:

$$F(k,h)=zk^{\theta}h^{1-\theta}. \tag{18}$$

Physical capital depreciation rate is set at $\delta_k = 6\%$. The $\boldsymbol{z}$ parameter of the production function, the relative price of physical capital p_k (which jointly determine the wage rate of human capital $\boldsymbol{w}$) and the early childhood human capital production function parameter h_B are chosen arbitrarily at such a level that the z_h parameter of the human capital production function gives model implications matching empirical data. The calibrated value of z_h is equal in all countries – in this way the model presumes that the ability to learn is the same in all countries.

Demographic parameters are taken as follows. The age when children are born (all children are born at the same time) is set to $B = 25$ in all countries. The retirement age is the smaller of the two numbers $\{64, T\}$, where T is the length of life in the considered country. T and f (the natural logarithm of the average number of children) are taken at their actual values.

The remaining parameters of the model are calibrated to match the following observations for the US economy: capital's share of income of 0,33; capital – output ratio of 2,52; earnings at age R to earnings at age 55 of 0,8; earnings at age 50 to earnings at age 25 of 2,17; years of schooling of 12,08; schooling expenditures as a fraction of GDP of 3,77; pre-primary expenditures per pupil relative to GDP *p.c.* of 0,14. This gives the following parameter values:

Table 1. The calibrated values of the Manuelli-Seshadri model parameters

Parameter	θ	r	δ_h	z_h	γ_1	γ_2	ν
Value	0,315	0,07	0,018	0,361	0,63	0,3	0,55

Source: Manuelli, Seshadri (2005).

The strategy of the Authors of the model was to choose the value of the z parameter (total factor productivity) to match the empirical value of GDP *p.c.*. Thus, the model gives predictions about the years of schooling and human capital accumulation related expenditures of a representative individual in the country under consideration.

Instead, we can take the empirical values of years of schooling and wage rate per unit of human capital and make predictions about GDP *p.c.* and the stock of human capital relatively to the USA.

4. Analysis of empirical findings for Poland and comparison to Manuelli and Seshadri's results

4.1. Predictions based on GDP p.c. calibration

We first present the predictions of the model for human capital in Poland. We take the actual value of years of schooling – $s = 10{,}75$[7] and demographic data for Poland ($f = 0$, the average length of life $T = 74$). We then calibrate the wage rate per unit

[7] In 2002 the educational structure of the Polish society was as follows (according to GUS): higher education 10,2%; secondary education 32,6%; vocational education 24,1%; primary education 28,2%; no education 4,9%. The types of education were ascribed, respectively, 17, 12, 11, 8 and 4 years of schooling and weighted average was calculated. An analogous procedure for 1988 gives $s = 10{,}03$.

of human capital $\boldsymbol{w}$ in such a way that the empirical value of GDP *p.c.* for Poland (13,5% of the American value) is a prediction of the model. It gives $w = 0,678$, in such units that $w = 1$ is the value of $\boldsymbol{w}$ for a representative individual in the USA. Therefore, a Pole with the same nominal stock of human capital as the American will get a salary equal to 67,8% of the American's salary. The difference in salaries is the measure of the Pole's lower quality of human capital.

On the actual labor market, a Polish immigrant gets 98% of the American-born worker's salary[8]. An important fact we have to take into consideration is that people who emigrate from Poland are better educated than an average Pole. As a result, they get a bigger salary in the USA than an average person educated in Poland would get. After taking this into account, the relevant quantity is 92,3% for a Pole with average education (of $s = 10,75$ years of schooling in Poland) as compared with an American with the same nominal education, age and sex. However, this is not the value of $\boldsymbol{w}$, since we have to address the fact that a lower value of $\boldsymbol{w}$ causes two effects. The first is that it lessens the salary of a person with the same stock of human capital (since the salary equals $wh(a)(1 - n(a)) - \pi x(a)$ where $\pi = 0,5$ is the part of the human capital accumulation expenditures incurred by the employer) and the second is that it reduces $h(a)$ for all ages a and thus decreases the average stock of human capital per person $\overline{h}$, which further lowers the salary. Taking it into account, the "empirical" value of $\boldsymbol{w}$ is equal to around 96,9%[9], which is much more than 67,8%, the value resulting from the calibration of the Polish GDP *p.c.* We will discuss this issue later and give some hypotheses about its origin. In this subsection, we will assume $w = 67,8\%$ and examine the model implications.

The following figures present the growth of the stock of human capital, the time allocated to human capital formation and expenditures on market goods allocated to human capital accumulation for a representative inhabitant of Poland, as compared with a representative inhabitant of the USA.

The paths of all quantities under consideration are the rescaled paths for the USA (it can be seen simply by looking at the left chart and the dashed line on the right chart). Having the appropriate paths for Poland and the USA on the same chart illustrates the relative differences.

The stock of human capital for an average Pole of any age equals roughly one fifth of their American peer's stock. When one calculates the average stock of hu-

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[8] This value and the following values according to: Hendricks (2002).

[9] It can be shown by integrating $wh(a)(1 - n(a)) - \pi x(a)$ multiplied by $\phi(a)$ from $6 + s$ to R and dividing by the integral of $\phi(a)$ in the same limits of integration, which gives the average salary of a Polish immigrant on the American labor market (taking Polish demographic structure into account). This value should be compared to the analogous value for the USA, but with the value of $\boldsymbol{s}$ for Poland to take into consideration the fact that an average Pole spends less time in school. If the ratio of the two values is equal to 92,3%, then the value of $\boldsymbol{w}$ (around 96,9%) corresponds to the empirical difference of the average Pole's and American's with the same nominal education salaries.

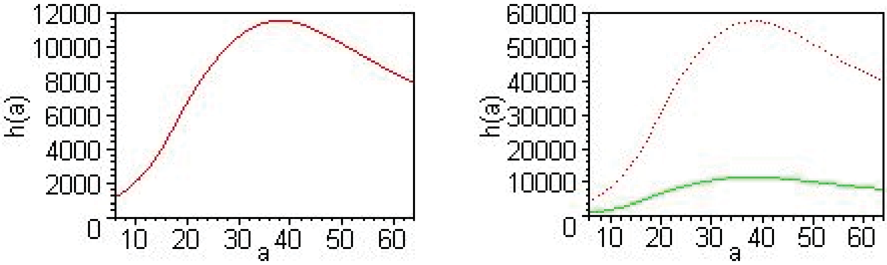

Figure 4. (left) The optimal path of human capital growth for a representative individual in Poland, aged 6 to 64; (right) A comparison of the path for representative inhabitants of Poland (solid line) and USA (dashed line)

Source: Own calculations

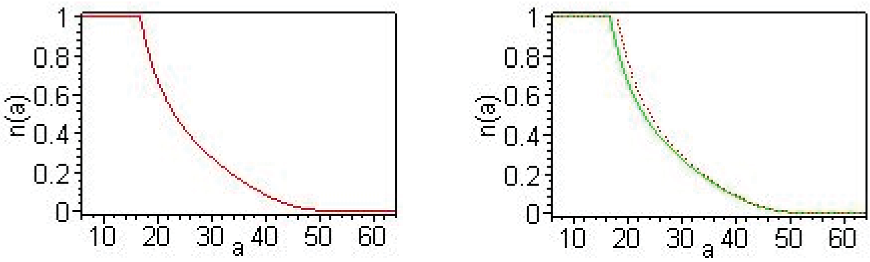

Figure 5. (left) The time allocated to human capital accumulation for a representative individual in Poland, aged 6 to 64; (right) A comparison of the path for representative inhabitants of Poland (solid line) and USA (dashed line)

Source: Own calculations

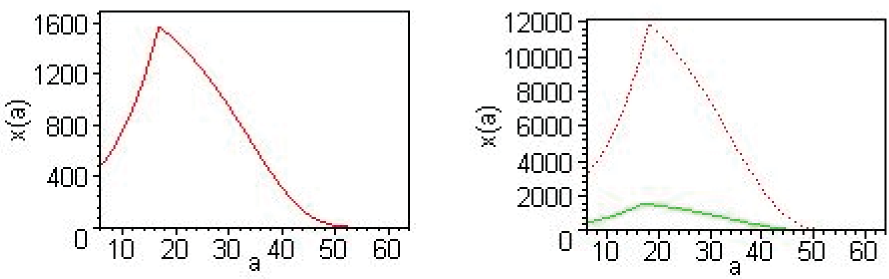

Figure 6. (left) Expenditures on market goods allocated to human capital accumulation for a representative individual in Poland, aged 6 to 64; (right) A comparison of the path for representative inhabitants of Poland (solid line) and USA (dashed line)

Source: Own calculations

man capital per person for Poland (Eq. (14)), one obtains approximately one fifth of the American's value as well. It suggests that the quality of human capital in Poland is much worse than in the USA. Despite the fact that the length of schooling is just around 10 percent shorter in Poland than in the USA, the ultimate stock of human capital is much smaller. Putting it another way, one year of learning in a Polish school equips students with much more modest amount of units of human capital, than one year of learning in an American institution.

An American, who finishes education on average more than a year later than a Pole, devotes slightly more time to human capital formation than his Polish peer. As a consequence, the working period is somewhat shorter for the American. However, the difference is minor and does not virtually matter from the point of view of the stock of human capital.

The differences are substantial, however, when it comes to expenditures on market goods allocated to human capital formation. An average Pole spends typically six times less than his American counterpart, which results from the lower quality of the Pole's human capital, since lower quality means meagrer stock of human capital and thus lower wage. The lower wage, in turn, leads to smaller expenditures on human capital formation related market goods. This effect brings about the growth of the absolute disparity between Polish and American stocks of human capital and expenditures on market goods allocated to human capital build-up.

If we want to compare the total factor productivity (TFP) and the physical capital-human capital ratio (κ) in Poland and the USA, we have to find the relative price of physical capital in Poland (Eq.(15)). The product $p_k(r + \delta_k)$ is equal to the marginal productivity of physical capital (right-hand side of Eq.(15)). In the USA $r = 7\%$, $\delta_k = 6\%$, so the price of capital equals ($p_k = 1$) $r + \delta_k = 13\%$. In the case of Poland, we will check the model predictions for three values of p_k: $p_k = 1$ (no differences in the price of physical capital – we isolate the role of human capital), $p_k = 1{,}2$ (assuming that physical capital in Poland is 20% more expensive than in the USA seems fairly reasonable) and $p_k = 2{,}3$ (it corresponds roughly to the $r \approx 23\%$ interest rate in Poland around the year 2000). The outcome of this experiment is shown below (Table 2).

Table 2. TFP and physical capital-human capital ratio in USA and Poland for different values of the price of physical capital p_k, $w = 67{,}8\%$

Country	p_k	TFP	κ
USA	1	1	3,537
Poland	1	0,766	2,398
	1,2	0,811	1,999
	2,3	0,996	1,043

Source: Own calculations.

The assumption of identical price of physical capital in Poland and the USA leads to a low value of total factor productivity in Poland, approximately 77% of the American value[10]. Physical capital is less widely used in Poland than in the USA. The value of $p_k = 1,2$ gives similar values of TFP and κ. Taking the empirical value of the interest rate as the price of physical capital results in identical values of TFP in Poland and the USA. Thus, explaining the difference in GDP *p.c.* between Poland and the USA does not compel us to take any difference in TFP – the joint effect of human capital and the price of physical capital suffices to match the observed difference. In such case, the stock of physical and human capital in Poland is practically the same.

It is worthwhile to assess the situation of Poland in comparison with other countries analyzed by Manuelli and Seshadri (Table 3).

Table 3. Comparison of the empirical values for the countries analyzed by Manuelli and Seshadri, grouped according to GDP *p.c.* The columns are: years of schooling (*s*), average length of life (*T*), average number of children per person ($e^f/2$) and the relative price of capital (p_k)

Decile	GDP *p.c.* (relative to USA)	s	T	$e^f/2$	p_k
90–100	0,921	10,93	78	0,85	1,02
80–90	0,852	9,94	76	0,90	1,11
70–80	0,756	9,72	73	1,00	1,06
60–70	0,660	8,70	71	1,20	1,04
50–60	0,537	8,12	69	1,35	1,52
40–50	0,437	7,54	64	1,60	1,77
30–40	0,354	5,88	57	2,05	1,56
20–30	0,244	5,18	54	2,50	1,93
10–20	**0,146**	**4,64**	**51**	**2,70**	**2,11**
0–10	0,052	2,45	46	3,10	2,78

Source: Manuelli, Seshadri [2005].

The Authors of the model divided the countries according to their GDP *p.c.* and calculated the average length of schooling, the relative price of physical capital and the demographic parameters. The Polish GDP *p.c.* is low and places Poland in the penultimate decile of countries according to this criterion. The average length of schooling in countries with comparable GDP *p.c.* equals 4,64 (for Poland $s = 10,75$), the average length of life 51 (in Poland T = 74), the average number of children

[10] It is worth emphasizing that a 23% difference in TFP which is needed to explain the difference in GDP *p.c.* is relatively small. Some researchers postulate that differences in TFP are the main source of cross-country differences in GDP *p.c.*

per person 2,7 (in Poland less than 1) and the average relative price of capital 2,11 (similarly in Poland if we assume the actual value of the interest rate in 2000). The cited parameters situate Poland among countries whose GDP *p.c.* equals roughly 80-90% of the American GDP *p.c.*. This seems to suggest that the mechanism which leads to the low value of the Polish GDP *p.c.* is different in Poland than in other countries with comparable GDP *p.c.* but completely disparate other parameters. This hypothesis is supported by the fact that the "calibrated" value of the wage rate per unit of human capital is well below the "empirical" value. We infer that one of the consequences of the recent economic and political transformation in Poland is the lack of market equilibrium and market structure inefficiencies, which result in lower (than resulting from Eqs. (15)-(17)) values of GDP *p.c.* despite a relatively high stock of human capital. We will comment on this hypothesis in the next subsection.

4.2. Analysis of sensitivity of the Manuelli-Seshadri model to the wage rate per unit of human capital

Let us now analyze how changing the value of $\boldsymbol{w}$ affects the predictions of the model concerning the stock of human capital. We will examine the values of $\boldsymbol{w}$ ranging from 65% to 100%, paying special attention to the „empirical" value for Poland of 96,9%. We assume $p_k = 1$, unless otherwise mentioned.

The following table presents the values of the stock of human capital at age 6 and 6+s along with the average stock of human capital per person for a representative individual in Poland, calculated for various values of $\boldsymbol{w}$. The data for the USA is given for comparison.

The wage rate per unit of human capital, a measure of the quality of human capital, strongly influences the relevant data. In case the wage rate is equal in Poland and the USA, the aforementioned stocks of human capital are similar and somewhat bigger in Poland than in the USA, which arises from the fact that an average Pole's working period is a little longer than the American's. In the "empirical" case (i.e. $w = 96{,}9\%$), the representative individual in Poland reaches 92% of the representative American's stock of human capital, which results almost entirely from the better quality of the American's human capital, the difference in the length of schooling being negligible. We will later see how such value of $\bar{h}$ affects GDP *p.c.*. For lower values of $\boldsymbol{w}$, the differences in the stock of human capital (at any age) are immense. The wage rate per unit of human capital elasticity of the stock of human capital is very high, equal to 4,29, which means that a one-percent increase in the wage rate per unit of human capital leads to 4,29-percent rise in the stock of human capital. The following chart illustrates this feature.

Let us now analyze the influence of the wage rate per unit of human capital on GDP *p.c.*, total factor productivity and the physical capital-human capital ratio (κ)

Table 4. The dependence of the stock of human capital at age 6 and 6+*s* and the average stock of human capital per person for a representative individual in Poland on the wage rate per unit of human capital. The data for USA are given for comparison

Country	*w*	*h*(6)	*h*(6+*s*)	$\bar{h}$
USA	**1**	**4657,98**	**25115,75**	**40304,40**
Poland	0,65	936,47	4077,53	6700,03
	0,678	**1122,00**	**4885,35**	**8027,41**
	0,70	1286,55	5601,85	9205,42
	0,75	1729,19	7529,13	12371,58
	0,80	2280,16	9928,15	16313,54
	0,85	2956,67	12873,79	21153,71
	0,90	3777,37	16447,25	27025,48
	0,923	4208,79	18325,73	30112,12
	0,95	4762,37	20736,07	34072,70
	0,969	**5184,19**	**22572,75**	**37090,65**
	1	5933,26	25834,31	42453,05

Source: Own calculations.

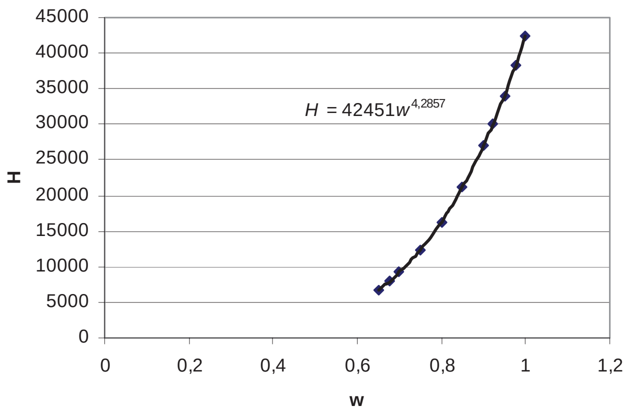

Figure 7. The dependence of the average stock of human capital per person (*H*) on the wage rate per unit of human capital (*w*) for Poland
Source: own calculations

(Table 5), as well as the influence of the relative price of physical capital on TFP and κ for the „empirical" value of w = 96,9% (Table 6).

The dependence of GDP *p.c.* on the wage rate per unit of human capital is also meaningful. Were the wage rates equal in Poland and in the USA, the Polish GDP *p.c.* would be higher than the American, as an effect of the slightly bigger average stock of human capital per person. The „empirical" value of ***w*** (96,9%) leads to

Table 5. The dependence of GDP *p.c.*, TFP and the physical capital-human capital ratio on the wage rate per unit of human capital. The data for USA given for comparison. GDP *p.c.* and TFP relative to USA

Country	w	GDP *p.c.*	TFP	κ
USA	1	1	**1**	**3,537**
Poland	0,65	0,108	0,744	2,299
	0,678	**0,135**	**0,766**	**2,398**
	0,70	0,160	0,783	2,476
	0,75	0,230	0,821	2,653
	0,80	0,324	0,858	2,83
	0,85	0,446	0,894	3,007
	0,90	0,604	0,930	3,184
	0,923	0,690	0,946	3,265
	0,95	0,803	0,965	3,360
	0,969	**0,892**	**0,979**	**3,428**
	1	1,053	1	3,537

Source: Own calculations.

Table 6. TFP and the physical capital-human capital ratio in USA and Poland for various values of the relative price of physical capital p_k, w=96,9%.

Country	p_k	TFP	κ
USA	1	1	3,537
Poland	1	0,979	3,428
	1,2	1,036	2,856
	2,3	1,272	1,490

Source: Own calculations.

89,2% of the American GDP *p.c.* for Poland, as a consequence of the 5-percent lower stock of human capital in Poland. The values of ***w*** of 65-70% correspond to GDP *p.c.* equal around 15% of the American GDP *p.c.* The wage rate per unit of human capital elasticity of GDP *p.c.* is 5,29, even higher than the elasticity of the stock of human capital. The following chart illustrates the Figure 8.

The dependence of TFP and the physical capital-human capital ratio on the wage rate per unit of human capital is linear. For lower values of ***w***, TFP at 80% of the American value is needed to match the observed GDP *p.c.* However, if we let the relative price of physical capital be higher in Poland than in the USA, much smaller difference in TFP is required to explain the empirical value of GDP *p.c.*

At the „empirical" value of w = 0,969, the assumption of equal prices of physical capital in Poland and the USA leads to similar values of TFP and κ for Poland and the USA. When the price of physical capital is 20 percent higher in Poland, the

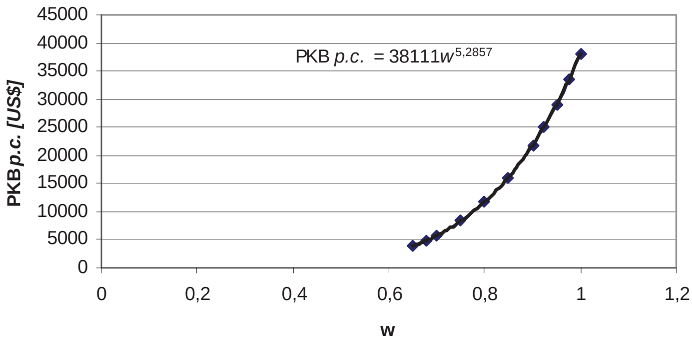

Figure 8. The dependence of GDP *per capita* (in US dollars) on the wage rate per unit of human capital for Poland (*w*)
Source: Own calculations

model equilibrium requires slightly higher values of TFP and lower κ. The price of capital which corresponds to the actual value of interest rate in Poland around the year 2000 leads to a 27-percent higher TFP in Poland than in the USA which is implausible, because it would mean a much higher level of technology in Poland than in the USA, the country commonly described as the world technology leader.

We have shown that the relevant variables of the Manuelli-Seshadri model strongly depend on the wage rate per unit of human capital. Earlier, we have stated that the „empirical" value of ***w*** (96,9%) is much higher than the „calibrated" value (67,8%). If we let w = 96,9%, the model is unable to match the actual value of GDP *p.c.* If we let w = 67,8%, the model places Poland among countries whose average inhabitant has three children (six per family) and dies at age 51, having spent 4,5 years at school. However, the predicted value of GDP *p.c.* agrees then with the empirical value.

Therefore, we encounter a paradox: the low GDP *p.c.* in Poland suggests poor quality of human capital in Poland, but then, how can we explain the fact that the Polish immigrants get such high salaries, just a little shy of the American natives' (with similar nominal education) salaries, if their human capital is of such low quality? Assuming that a Pole gets 96,9% for one unit of his human capital instead of 67,8% he should get means that we undermine the role of free market as a perfect measure of value. Possibly, some political or social factors make it impossible to offer such low salaries to Polish immigrants, since the employers would be accused of abusing them and treating them as a cheap workforce. In other words, the representative Pole's human capital is of much worse quality than the representative American's with identical nominal length of schooling, but the salaries do not fully capture this difference in quality.

However, the values of ***w*** equal to 96,9% and 67,8% give salaries, respectively, at 92,3% (empirical value given by Hendricks) and merely around 14% of the average American's salary. Is the market so unreliable that it values the work of the average

Polish immigrant at 92% of the average American's value instead of 14% - almost sevenfold more than it is worth, taking the low quality of the Polish immigrant's human capital into consideration? It seems impossible that the market inefficiencies could be so high.

Presumably the value of ***w*** which mirrors the actual quality of human capital in Poland[11] is closer to the "empirical" value of 96,9%, not the shockingly low 67,8%. Assuming this we get the stock of human capital for Poland just shy of the American and almost equal values of the Polish and American GDP *p.c.* The model is thus unable to predict the actual value of the Polish GDP *p.c.* correctly. The reasons for the divergence of the model value (close to American value) and the actual value (around 13,5% of the value for the USA) have to be looked for outside the model.

The model fails to take into consideration the fact that the market economy in Poland is relatively young – only a dozen years or so have passed since its launch, which appears to be fairly important. Firstly, the time span might have been too short for equilibrium to be achieved. In such case, equations (15)-(17) should be modified to allow for this effect. Secondly, the structure of the Polish economy is not yet human capital-oriented, i.e. relatively high stock of human capital in Poland is not fully used in production. The high stock of human capital is a prediction of Manuelli and Seshadri's model for the US economy with Polish immigrants. The American economy can be well described within this framework, so we may view the prediction of the high stock of human capital in Poland as fairly reliable. Assuming the high stock of human capital in Poland, the predictions of the model are quite optimistic – when the factors related to the young age of the Polish free market economy cease to have an effect and the human potential is taken advantage of, the Polish GDP *p.c.* should reach a level close to the American or Western European level.

5. Conclusion

In this paper we have presented the Manuelli-Seshadri model. The model gives a special role to human capital and its two aspects: the quantity and the quality. We have analyzed the process of calibration of the model parameters and applied the model to the American and Polish economies. We have compared the stock of human capital, expenditures and time allocated to human capital formation, as well as the wage rate per unit of human capital and GDP *p.c.* in both countries. We have shown the conclusions that arise from the model for Poland and the comparison of

[11] Poland is often quoted to be among countries with relatively high quality of schooling, comparable or only a little worse than in Western Europe or USA.

the Polish economy with economies of other countries (from all around the world) analyzed by Manuelli and Seshadri. We have performed the wage rate per unit of human capital sensitivity analysis, since the wage rate is a measure of the quality of human capital, a concept introduced by the Authors of the model. The concept makes it possible to match empirical data quite well.

Our results are as follows. The wage rate per unit of human capital, calibrated in such a way that the model matches the empirical level of Polish GDP *p.c.*, is very low. It is tantamount to catastrophically inferior quality of the representative individual's human capital in Poland and situates Poland among the poorest countries of the world. The empirical value of the wage rate per unit of human capital $\boldsymbol{w}$ (based on the data from Hendricks (2002)) appears, however, to be close to the American value. The resulting value of the Polish GDP *p.c.* is almost as high as the American GDP *p.c.* Despite this unrealistic feature, the model predictions concerning human capital seem to be plausible.

It follows, however, that the model is unable to explain the low value of Polish GDP *p.c.* A possible cause of this is the still-lasting economic transition in Poland and almost fifty years of non-free market economy, as the sources of two basic phenomena. Firstly, the Polish economy is probably rather far away from equilibrium, understood in terms of the neoclassical steady-state equations for physical capital and labor. Secondly, the structure of the Polish economy is not well-suited to make use of the fairly big stock of human capital in Poland. Both of these phenomena, especially the second one, lower the value of the Polish GDP *p.c.* below its potential value, predicted by the Manuelli-Seshadri model. Taking the phenomena into consideration will probably enable its more universal application and offer a better explanation of the cross-country differences in output per worker.

It seems, however, that the model conclusions concerning the stock of human capital and other related quantities are very interesting and apposite. The model explanation of the relation of the quantity and quality of human capital to GDP *p.c.* is satisfactory for many countries.

The Manuelli-Seshadri model is relatively simple, which facilitates its modification in the direction of a better fit to empirical data. Firstly, the model does not take into consideration the effects of economic transformations in countries like Poland. Secondly, the only measure of the quality of human capital is the wage rate per unit of human capital. It seems reasonable to try to broaden this concept. Finally, the model does not implement technological progress, although no human capital growth on a world scale would be possible without the developing science and technology. Human capital and technological progress are strongly interrelated, so their simultaneous analysis would be highly justified.

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